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Translated by E.L.S.

J. Appl. Maths Mechs Vol. 56, No. 4, pp. 545–558, 1992
Printed in Great Britain.
0021–8928/92 $24.00 + .00

EXTREMAL PROBLEMS OF HEAT TRANSFER TO THREE-DIMENSIONAL BODIES AT HYPERSONIC SPEEDS†

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(*Received* 27 *January* 1991)

The design of shuttle-like hypersonic spacecraft [70, 77] gives rise to the problem of investigating the spatial configurations that are optimum from the point of view of thermal heating and other characteristics, which enable the weight of the required thermal protection to be reduced. The problem of optimizing the weight of thermal protection depends on many parameters and has not yet been solved in a rigorous mathematical formulation. Approximate formulations of the optimization problem have therefore been considered, the solution of which has enabled axisymmetrical optimum shapes of bodies to be obtained with the minimum convective [43, 61, 73] and radiation [35–37, 58, 60] heat fluxes. It is known from attempts to solve variational problems of a body with minimum drag [29–31, 51–53, 62], that the transition to essentially three-dimensional configurations enables a reduction in the drag to be achieved compared with axisymmetrical bodies. A similar situation should obviously also occur when optimizing the shape of a body for heat flux. In this paper we present for the first time variational problems for finding the optimum shape of three-dimensional bodies of minimum overall thermal heating when moving along an incoming trajectory. In papers by other authors [21, 29–31, 48, 51–53, 62] the problems of determining the three-dimensional optimum aerodynamic shapes from the point of view of the minimum wave or total drag were considered. A brief review is given of research which has been done to determine the convective and radiation heating of three-dimensional bodies and the fundamental formulas for the wave drag, the fraction drag, and the convective and radiation fluxes to three-dimensional bodies moving in dense layers of planetary atmospheres are presented. The formulas depend explicitly on the conditions of entry into the atmosphere of the planet and on the geometry of the body, which enables variational problems to be formulated on determining the three-dimensional shape of the body from the conditions for minimum overall heating (convective and radiation) of the surface along the trajectory of motion.

1. FORMULATION OF VARIATIONAL PROBLEMS ON CHOOSING THE OPTIMUM SHAPE OF THREE-DIMENSIONAL BODIES OF MINIMUM OVERALL HEAT TRANSFER

CONSIDER the motion of a three-dimensional body in a planetary atmosphere along a plane trajectory at a hypersonic velocity acted upon by a lift force, a frontal drag, a gravity force and a

† *Prikl. Mat. Mekh.* Vol. 56, No. 4, pp. 643–657, 1992.

centrifugal force. We will neglect the effect of the reactive force due to the loss of mass. The conditions of balance of the forces along the tangent and normal to the trajectory can then be written in the form [50]

$$\frac{1}{2}\rho v^2 C_D S + M\frac{dv}{dt} = Mg\sin\gamma \tag{1.1}$$

$$\frac{1}{2}\rho v^2 C_L S_w + Mv\left(\frac{d\Phi}{dt} + \frac{d\gamma}{dt}\right) = Mg\cos\gamma \tag{1.2}$$

Here M, v, S, and S_w are the mass, velocity, characteristic area and the wetted area of the body, respectively, ρ is the gas density at the height z_*, t is the time, g is the gravitational acceleration, γ is the angle of entry into the planetary atmosphere, Φ is the angular distance, and C_D and C_L are the coefficients of total drag and the lift force.

Equations (1.1) and (1.2) cannot be solved analytically. A wide range of problems exists, however, [50] for which one is justified in simplifying Eqs (1.1) and (1.2).

Consider the case of non-vertical entry into the atmosphere with a negligibly small centrifugal force. For angles of entry into the Earth's atmosphere of $\gamma > 10°$ the centrifugal forces can be ignored at hypersonic speeds of entry [50], in which case Eqs (1.1) and (1.2) take the form

$$\frac{d\gamma}{dt} = -\frac{\rho g v K_1}{2\beta}, \quad \frac{dv}{dt} = -\frac{\rho g v^2}{2\beta}, \quad \frac{dz}{dt} = -v\sin\gamma,$$
$$\beta = \frac{Mg}{C_D S}, \quad K_1 = K\frac{S_w}{S}, \quad K = \frac{C_L}{C_D}, \tag{1.3}$$

where β is a ballistic factor and K is the aerodynamic quality. From (1.3) we obtain

$$\frac{1}{v}\frac{dv}{dt} = K_1^{-1}\frac{d\gamma}{dt}, \quad \gamma = \gamma(z_*). \tag{1.4}$$

For an isothermal atmosphere $\rho = \rho_0 \exp(-\lambda z_*)$, where ρ_0 is the density of the atmosphere at the level for the planetary surface and λ^{-1} is the height scale for the density.

Equations (1.3) are solved for the following initial conditions: $t = 0$, $z_* = z_0$, $v = v_0$, $\gamma = \gamma_0$. Integrating (1.4) we can obtain

$$v = v_0 \exp\{-K^{-1}[\gamma_0 - \gamma]\} \tag{1.5}$$

From (1.3) and expression (1.5) we obtain when $\lambda z_0 \gg 1$

$$\gamma = \arccos\left[\frac{\rho_0 g K_1}{2\beta\lambda} e^{-\lambda z_*} + \cos\gamma_0\right] \tag{1.6}$$

When $K_1 \ll 1$ we have $\gamma = \gamma_0 = \text{const}$.

Other analytical solutions of (1.1) and (1.2) are given in [50].

If the heat spreads along the normal to the surface of the body, the equation describing the heating of the spacecraft [61] has the form

$$\frac{dH}{dt} = \frac{1}{2}\rho v^3 C_H S \equiv Q, \quad C_H = C_C + C_R \tag{1.7}$$

Here H is the total heat absorbed by the surface of the body when it moves along the trajectory, and C_H is the overall heat-transfer coefficient, which consists of the coefficients of radiation heat transfer C_R and convective heat transfer C_C.

Integrating (1.7) we obtain the total amount of heat which goes to raise the body temperature along the whole of its trajectory or in the range of heights

$$H = \int_0^t \frac{\rho v^3 C_H S}{2} dt = \int_{v_k}^{v_0} \frac{M C_H}{C_D} v\, dv \tag{1.8}$$

$$H=\int_{z_k}^{z_0} \frac{MC_H}{C_L} \frac{S}{S_w} \frac{d\gamma(z_*)}{dz_*} v^2(z_*)dz_* \tag{1.9}$$

Here the expressions $v = v(z_*)$ and $\gamma = \gamma(z_*)$ are given by (1.5) and (1.6), respectively, z_k is the final height of launching and v_k is the final speed of the body.

Expressions (1.8) and (1.9) for known gas-dynamic relations for the coefficients C_H, C_L and C_D as a function of the geometry of the body enable us to formulate the variational problem of determining the body shape from the conditions for least overall heat flux along the trajectory to the body surface.

Consider entry into the atmosphere along a ballistic trajectory ($C_L = 0$). Because of the considerable drag forces which arise at hypersonic speeds of entry along a ballistic trajectory in the atmosphere, when determining the velocity of motion one can neglect the effect of gravitational forces [73] with an error of not more than a few percent. Moreover, neglecting the centrifugal forces, we obtain from (1.1) and (1.2)

$$M\frac{dv}{dt} = -\frac{1}{2}\rho v^2 C_D S, \quad \frac{dz_*}{dt} = -v \sin\gamma \tag{1.10}$$

Integrating these equations with the initial conditions we obtain

$$v = v_0 \exp(-a_1 e^{-\lambda z_*}), \quad a_1 = \rho_0 g/\lambda 2\beta \sin\gamma$$

where a_1 is the trajectory parameter.

The overall aerodynamic heating of the body H along the trajectory is given by (1.8). The variational problem can then be formulated as follows.

It is required to obtain the equation of the body shape $z = f(x, y)$ for which the overall heat flux H to the body along its trajectory is a minimum, and which satisfies the specified isoperimetric and boundary conditions.

One can specify as the isoperimetric conditions the volume of the body V, the base area S or the wetted area S_w [62]

$$V = 2\iint_S f(x,y)\,dx\,dy, \quad S = \iint_S dx\,dy \tag{1.11}$$
$$S_w = \iint_{S_w} \sqrt{1+f_x'^2+f_y'^2}\,dx\,dy$$

Usually one takes as the boundary condition the equation of the leading edge of the body, namely,

$$y = 0, \quad z = \varphi_*(x) \tag{1.12}$$

In the case of entry into a planetary atmosphere with a lift force one can consider, in addition, limitations on the lift force and on the aerodynamic quality

$$\begin{gathered}\iint_S \frac{f_y'\,dx\,dy}{[1+f_x'^2+f_y'^2]} = \frac{LS}{2} = \text{const} \\ \iint_S \frac{2f_y'\,dx\,dy}{[1+f_x'^2+f_y'^2]} = K\iint_S [2[1+f_x'^2+f_y'^2]^{-1} + C_f\sqrt{f_x'^2+f_y'^2}]\,dx\,dy\end{gathered} \tag{1.13}$$

Here L is the specified value of the lift force and K is the specified aerodynamic quality.

We can consider as an additional limitation, limitations on the local overall heat flux q [37]: $q \leq q^*$, $q = q^c + q^R$, where q^* is the specified limiting value of the flux.

If the bodies possess homothetic properties, i.e. they satisfy the condition that each cross-section of the body normal to the z axis is geometrically similar to the cross-section in the plane of the base, the surfaces of the bodies can be represented by an equation in a cylindrical system of coordinates $(r, \varphi, z): r = f(z)\Phi(\varphi)$, where the function $f(z)$ and $\Phi(\varphi)$ respectively define the longitudinal and transverse contour of the body. The integral for the overall heat flux H can then be minimized with the following limitations [62]:

for a specified area of the base

$$S = \frac{1}{2} \int_0^{2\pi} \Phi^2(\varphi)\, d\varphi f^2(l) \tag{1.14}$$

or the conditions for the contour to be closed

$$\Phi(0) = \Phi(2\pi) = \Phi_0 \tag{1.15}$$

or the boundary condition for $f(z)$:

$$f(0) = 0, \quad f(l) = f_0 \tag{1.16}$$

where l is the length of the body along the z axis.

From the general formulation we obtain the problem of finding the shape of a three-dimensional body from the point of view of the minimum overall convective heat flux from the surface $Q_C(C_R = 0,\ C_H = C_C)$ or the overall radiation heat flux $Q_R(C_C = 0,\ C_H = C_R)$ along the trajectory of the body.

In the variational problem for the functional H, it follows from (1.8) and (1.9) that the trajectory and shape of the body are interrelated. In a number of cases [36] we can separate this general problem and consider the optimization of the shape of the body in the neighbourhood of the maximum-heating point of the trajectory and in the case of a specified trajectory. In this case the ballistic factor (β = const) and the aerodynamic quality (K = const) are specified.

Using the expressions obtained we can also investigate other optimization problems including multiparametric ones. For example, it is required to obtain the shape of the body such that the following conditions are satisfied:

$$H = \min, \quad T = \min$$

$$T = \int_{z_k}^{0} \frac{dz_*}{v \sin\gamma(z_*)}, \qquad \left| M \frac{dv}{dt} \right| = \min$$

i.e. to ensure minimum heating, minimum time of motion along the trajectory and the least overloading.

2. FORMULAS FOR CALCULATING THE CONVECTIVE HEAT FLUXES AND THE FRICTION DRAG FOR THREE-DIMENSIONAL BODIES

For the case of motion in the Earth's atmosphere with velocities of the order of the first cosmic velocity, convective heat flux between the gas and the surface of the body around which flow occurs make the main contributions to the overall aerodynamic heating [59].

When obtaining the fundamental formulas for convective heat flux at high Reynolds numbers, we can use approximate methods of calculating the three-dimensional boundary layer in a compressible gas [1, 38, 66, 74, 81]. The well-known "axisymmetrical analogy" method [74] enables us to calculate the parameters of the boundary layer independently along each inviscid streamline on the surface of a three-dimensional body. Using the hypothesis of local similarity with a plate as the approximate method of calculating the local convective heat flux q_w^c we have

$$hq_w^c = \varphi_1 \left[\int_0^s \varphi_1 ds \right]^{-1/2}$$

$$\varphi_1 = \rho_0 \mu_0 A_0^2 (h_0 - hw)^2 \left[1 + \frac{p^b}{1{,}22k_0 + 0{,}4} \right]^{\omega - 1} v_l p h^2$$

$$p = p_e / p_0, \quad A_0 = 0{,}332 \mathrm{Pr}^{-2/3}, \quad b = (\gamma - 1)/\gamma \tag{2.1}$$

Here h is the analogue of the axisymmetrical radius, the integration is carried out along the inviscid streamline, v_e and p_e are the velocity and pressure on the outer edge of the boundary layer,

h_w is the enthalpy at the temperature of the body surface, h_r is the enthalpy of the heat-insulated surface, we assume a dependence of the coefficient of viscosity on the temperature of the form $\mu \sim T^w$, γ is the ratio of the specific heat capacities, the subscript "0" denotes parameters outside the direct shock wave, $k_0 = T_w/T_0$ is a quantity that is small by assumption, and h_0 is the stagnation enthalpy of the flow.

If q_w^c—the heat flux at a given point of the body surface—is known, and S is the body surface area, the overall heat flux to the body surface is

$$Q_c = \iint_S q_w^c \, dS \tag{2.2}$$

Using this method, the heat transfer to an infinite cylinder with slip, a cone, and also the surface streamlines in the neighbourhood of the leading critical point were considered in [1] in a system of coordinates connected with the inviscid streamlines on the body surface and neglecting secondary flows.

In [66] this method is justified by an asymptotic analysis with respect to a small parameter connected with the curvature of the inviscid streamlines on the body surface. Comparison of the calculations of heat fluxes on the surface of a spherically blunt cone with a semi-aperture angle of 10° at an angle of attack of 10° with experimental data showed good agreement.

The "axisymmetrical analogy" method was used in [75] to find the heat transfer to the surface of a typical configuration of an orbital stage at the angle of attack. Comparison of the calculations with experimental data shows good agreement in the heat fluxes on the windward side of the body at the angle of attack. The heat transfer to the surface of an orbital stage is also investigated in [81]. Unlike [66], the velocity field is found from the accurate equations using a calculation of three-dimensional inviscid flows when there are equilibrium or non-equilibrium chemical reactions. The accuracy and limits of applicability of the method are established by comparison with experimental data and with the solution of the Navier–Stokes equations for the simplest flows.

The particular problem of choosing the optimum trajectory and the body shape which reduces thermal heating is considered in [77]. The motion of an axisymmetrical spherically blunt cone at the angle of attack is investigated. At first, trajectory parameters of the motion of the body are considered, and then the optimum angle of attack is specified. The results of the calculations in [77] were checked experimentally in a shock tube. The variational problem of determining the shape of the three-dimensional body was not considered directly in this paper.

The problem of determining the shape of a body of minimum convective heat flux having an elliptic cross-section with a constant ratio of the semi-axes was considered in [11]. The solution was obtained in parametric form for specified dimensions of the body for laminar flow with constant flow parameters. Lees' formula [56] was used to determine the distribution of the heat flux over the body surface, the overall heat flux was found by integration over the generatrix of the elliptic body, but the three-dimensional nature of the external inviscid flow was unjustifiably neglected. The solution of the problem in [11] is a considerably simplified particular example of the determination of a three-dimensional body that is optimum with respect to heat transfer at a given point of the trajectory.

An approximate method was developed in [22] for the rapid calculation of heat fluxes on bodies of complex shape. The results of the calculation of three-dimensional inviscid flow are used to determine inviscid streamlines on the body surface along each of which the heat fluxes are calculated approximately, independently of the others. It was shown that one can correctly calculate the heat transfer both under conditions characteristic for wind tunnels and under actual flight conditions. A feature of this method is the possibility of correctly calculating heat fluxes on the wing of an orbital stage. Note that configurations which include wings with longitudinal overflow are of interest in the development of hypersonic manoeuvring aircraft [47].

An investigation of the flow around three-dimensional configurations with wings may lead to the discovery of new aerodynamic effects connected with heat transfer [64, 77, 82], which must be taken into account in optimum design.

A study of the heat transfer in the neighbourhood of the streamline of a three-dimensional body or an axisymmetrical body at the angle of attack is of particular interest since extremal heating occurs along such a line [56].

The specific heat flux per unit surface in the region of a streamline is

$$\begin{gathered} Q_c = 2\sqrt{\rho_0 \mu_0 v_\infty} A_0 (h_0 - h_w) J \\ J = I_1^{1/2}/I_2, \quad I_1 = \int_{z_l}^{z_f} y'^2 h^2 \, dz, \quad I_2 = \int_{z_l}^{z_f} h \, dz \end{gathered} \tag{2.3}$$

Here (z_i, y_i); (z_f, y_f) are the end points of the streamline $y = y(z) = f(z, 0)$.

The metric coefficient h is determined from the known geometry of the inviscid streamline. An equation for h was obtained in [75] on the assumption that the velocity field on the surface is determined by Newton's theory. This equation on the streamline has the form

$$\frac{h'(z)}{h(z)} = y'(z)\,k(z), \quad k(z) = \frac{f(z,0) - f''_{\varphi\varphi}(z,0)}{f^2(z,0)} \tag{2.4}$$

where k is the curvature of the line in transverse cross-sections (z = const) of the surface $r = f(z, \varphi)$. By integrating (2.4) we can determine the shape of the streamline from the known functions $h(z)$ and $k(z)$.

The variational problem of choosing the optimum contour of the streamline of a three-dimensional body and the curvature of the surface in the neighbourhood of this line for laminar hypersonic flow of a gas around a body was investigated in [43] using functional (2.3). The heat fluxes to optimum and non-optimum forms in the neighbourhood of the streamline were compared.

A simultaneous calculation of the heat flux on an orbital spacecraft and the characteristics of the thermal protection system were carried out in [64] using experimental data from the "Space Shuttle". The calculated and experimental values of the temperature and heat fluxes on the lower windward side of the surface of the fuselage of the orbiting spacecraft in the plane of symmetry and also on the lower surface of the wing in sections at 50% and 80% of the length of its span were compared. The results [64] show that the calculation somewhat exaggerates the heat flux on the windward side of the fuselage surface and underestimates that on the nose part, as well as on the wing surface in a section situated at a distance of 50% of the span.

We considered above approximate methods of investigating heat transfer in three-dimensional flow around bodies at high Reynolds numbers, when the flow separates into an inviscid flow and a boundary layer close to the surface. Approximate methods for calculating the flows when viscous effects are considerable over the whole shock layer, which occurs at low and moderate Reynolds numbers, were developed in [12–19]. Formulas were obtained in [16, 17] for determining the heat fluxes and the friction stress in the neighbourhood of a plane of symmetry of bodies around which flow occurs at the angle of attack for low and moderate Reynolds numbers using the approximate solution of the equations of the three-dimensional viscous shock layer, taking into account slip and the jump in temperature on the surface. The flow is investigated using a two-layer model of a viscous shock layer, similar to the model of axisymmetrical flow and assuming a thin perturbed region of flow using the method of successive approximations [13, 19, 57]. A formula is proposed in [16, 17] by means of which the calculation of the heat flux in the neighbourhood of a plane of symmetry of three-dimensional bodies around which flow occurs at the angle of attack is reduced to calculating the heat flux at an axisymmetrical critical point.

For flows of a uniform gas in the neighbourhood of a plane of symmetry of blunt bodies it has been established that for moderate and high Reynolds numbers the distribution along the surface of the heat flux, referred to its value at the stagnation point, ceases to depend on the Reynolds number. It also depends only slightly on the ratio of the specific heat capacities, the Prandtl number, the value of the exponent in the relation ($\mu \sim T^{\omega}$), the surface temperature (for a cooled wall) and also on whether the effects of slip and temperature jump on the surface are taken into account, and is mainly defined by the geometrical characteristics of the body.

To estimate the heat fluxes on a surface streamline of blunt bodies, axisymmetrical flow for a body formed by the rotation of a surface streamline around an axis parallel to the direction of the flow is sometimes used [76, 82]. However, this heat flux without corrections connected with the curvature of the surface may give considerable errors [18]. Unlike the axisymmetrical analogy used in boundary-layer theory [1, 74], the formula proposed in [18] is applicable not only at high Reynolds numbers but also at moderate and low Reynolds numbers. In [15] similar formulas were obtained for the stagnation point of a three-dimensional body, while in [19] the general case of three-dimensional flow around blunt bodies by a uniform viscous gas is considered over a wide range of Reynolds numbers at angles of attack and slip.

We will consider in more detail the formulas obtained in [19], for which we will consider steady-state three-dimensional supersonic laminar flow of a viscous gas around smooth blunt bodies at an angle of attack in a system of Cartesian coordinates XYZ connected with the body. The origin of coordinates O is placed at the stagnation point of the body. The Z axis coincides in direction with the velocity vector of the flow and the X and Y axes are chosen so that, at the stagnation point, the directions of the corresponding coordinate lines lie in the planes of principal curvature of the body. Suppose the body surface is specified in a Cartesian system of coordinates by the equation $z = f(x, y)$.

The equations of a three-dimensional thin viscous shock layer were derived in [57]. The system of equations obtained in Dorodnitsyn-type variables were solved in [19] by the method of successive approximations [13].

The Stanton number and the local coefficients of friction are given by the following formulas:

$$\mathrm{St} = \frac{q_w^c}{\rho_\infty v_\infty (h_\infty - h_w)}, \quad C_f^\alpha = \frac{\tau_w^\alpha}{\rho_\infty v_\infty^2} \tag{2.5}$$

Calculations carried out in [19], like the numerical solution of the system of equations of a three-dimensional viscous shock layer, showed that for $\mathrm{Re} \geqslant 100$ the distribution of the heat flux over the surface, related to its value at the stagnation point, calculated using the modified Rankine–Hugoniot relations on a shock wave and the boundary conditions on the body surface, which take into account the rate of slip and the temperature jumps, are practically identical with the corresponding distribution calculated using the ordinary Rankine–Hugoniot relations and the adhesion conditions and the specified temperature on the body. In the latter case, the formulas are simplified considerably and take the following form (the subscript "0" denotes the values of the corresponding quantities at the stagnation point):

$$\frac{\mathrm{St}}{\mathrm{St}_0} = \frac{q_w^c}{q_{w_0}^c} \cos^{3/2}\alpha \sqrt{\frac{H_*}{H_0 \lambda}}, \quad \cos\alpha = 1/\sqrt{g}$$
$$\lambda = 1 + \frac{4}{15} \frac{f_{11}'' f_1' + 2 f_{12}'' f_1' f_2' + f_{22}'' f_2'^2}{g^{3/4} H_*}; \quad f_1 = \frac{\partial f}{\partial x}; \quad f_2 = \frac{\partial f}{\partial y} \tag{2.6}$$

$$H_* = \frac{1}{2g^{3/2}} [f_{11}''(1 + f_2'^2) + f_{22}''(1 + f_1'^2) - 2 f_{12}'' f_1' f_2']$$
$$g = 1 + f_1'^2 + f_2'^2 \tag{2.7}$$

Here H_* is the mean curvature of the surface at the point considered.

Similarly a formula is obtained for the relative coefficients of friction for $\mathrm{Re} \geqslant 100$:

$$\frac{C_f^i}{C_{f_0}^i} = \sqrt{\frac{H_*}{g H_0}} \frac{(1 - 2/3a - 2\eta^i)}{(1 - 2/3a_0 - 2\eta_0^i)}$$
$$a^4(1 - \mathrm{Pr}) - a^3 + E^*(1 - a)^2 = 0, \quad E^* = g(1 - G_w)\mathrm{Pr}(\varepsilon \mathrm{Re}/H_*)^2$$
$$\eta^i = \begin{cases} a f_{11}''/(6g^2 H_*), & i = 1 \\ [a/(6H_*)]\{2H_* - f_{11}''/g - f_{122}^{(\mathrm{III})} f_1'/(g f_{22}'')\}, & i = 2 \end{cases} \tag{2.8}$$

A simple formula was proposed in [15] for calculating the heat flux at a three-dimensional critical point, which reduces to calculating the heat flux at an axisymmetrical critical point

$$\mathrm{St}_0(\kappa, \mathrm{Re}) = \mathrm{St}_{0c}(\mathrm{Re}_0), \quad \mathrm{Re}_0 = \frac{2\mathrm{Re}}{1 + \kappa} \tag{2.9}$$

Here St_0 is the Stanton number at the three-dimensional stagnation point, defined for the Reynolds number Re, St_{0c} is the Stanton number at the axisymmetrical stagnation point defined for the Re_0 number and κ is the ratio of the principal curvatures at the stagnation point $(0 \leqslant \kappa \leqslant 1)$.

It was shown in [15] that it is possible to use a simplified relationship between St_0 and St_{0c} for the range of Reynolds numbers considered

$$\mathrm{St}_0/\mathrm{St}_{0c} = \sqrt{1/2(1 + \kappa)} \tag{2.10}$$

Here St_0 and St_{0c} are calculated for the same Reynolds number Re.

Relations were established in [20] for recalculating convective heat fluxes to smooth three-dimensional bodies knowing the heat fluxes to the equivalent axisymmetrical bodies in the case of hypersonic flow, taking into account non-equilibrium chemical reactions in the shock layer and catalytic reactions on the body surface. These relations were obtained for the air flow around the body at Reynolds numbers of $10^2 < \mathrm{Re} < 10^6$, Mach numbers $M_\infty \to \infty$ and a ratio of the principal curvatures at the stagnation point from 0 to 1. The relations were derived by comparison with the results of numerical solutions of the problem of flow around ellipsoids.

Hence, to calculate convective heat flux to smooth three-dimensional bodies similar formulas have been developed [13–20], which can be employed to solve various optimization problems over a wide range of Reynolds numbers.

3. FUNDAMENTAL FORMULAS FOR CALCULATING RADIATION HEAT FLUXES TO THREE-DIMENSIONAL BODIES

On entering the upper layers of the atmosphere at velocities of the order of the second cosmic velocity or above, radiation may have a considerable effect on the flow of gas around the body, and the radiation heat fluxes are comparable with the convective heat fluxes or may even exceed them [35]. Problems of the flow of a hypersonic radiating gas around plane and axisymmetrical bodies and the heat transfer that occurs were considered in [18, 58, 59]. Problems of the three-dimensional flow of a radiating gas around bodies involve additional difficulties and have therefore not been investigated to such a great extent [8].

A numerical investigation of radiation heat transfer on three-dimensional bodies was carried out in [10, 23–25, 40–42, 65], using various methods of solution and assumptions regarding radiation transfer.

In these papers, a solution was obtained numerically only for a limited class of bodies and flow conditions and are not of universal form. Hence, it is important to set up formulas for determining the radiation heat fluxes to three-dimensional and axisymmetrical bodies using existing numerical results.

Three-dimensional supersonic flow of inviscid non-heat-conducting radiating air with a compressed layer around a blunt body was considered in [3–5] taking equilibrium chemical reactions into account. The radiation flux was calculated in the plane-layer approximation. The flow around a wide range of three-dimensional and axisymmetrical bodies (triaxial ellipsoids, paraboloids, spheres and bodies of power form) were investigated over a range of dimensions from 0.01 to 20 m, a range of velocities from 8 to 18 km/s and heights from 40 to 80 km in the Earth's atmosphere. The layer of vapour was not taken into account directly, but the attenuation of the radiation flux by this layer was modelled approximately. The results of numerous calculations [3–5, etc.] show that the relative distribution of the radiation heat fluxes over the surface of the bodies at fixed flight velocities depends only slightly on the dimensions and shape of the body and on the flight altitude.

For axisymmetrical bodies it can be approximated [3–5] by an analytical function of the angle of inclination of the body θ_w or the angle of inclination θ_s of the head shock wave to the flow. (The angle is taken between the external normal to an element of the surface and the velocity vector of the gas flow.) In the case of flow around three-dimensional bodies, as calculations show [3–5], universality has only been obtained in the dependence on θ_s. The values of θ_s in any cross-section of the three-dimensional shock layer depend very slightly on the characteristic dimensions of the body, the velocity and the altitude of flight, and can be obtained by solving the problem of the adiabatic flow of an ideal gas at any specified velocity of motion.

To determine the absolute values of the radiation fluxes to a body it is necessary to have data on their values at the critical point of blunting q_{w0}^R [3, 4]. The approximation formula for calculating the relative radiation heat fluxes to the body surface has the form [3–5]

$$\bar{q}_R(\theta_s) = \frac{q_w^R(\theta_s)}{q_{w0}^R} = \cos^n \theta_s, \quad n = 1.811 + \frac{1}{0.051\, v_\infty - 0.43} \tag{3.1}$$

Here v_∞ is the body speed in km/s.

For axisymmetrical flows we can use the approximate relationship [63]

$$\theta_s = \theta_w - \operatorname{arctg}[0.164 \sin\theta_w(\cos\theta_w + \sqrt{1 - 0{,}698 \sin^2\theta_w})] \tag{3.2}$$

For angles $\theta_w \leqslant \pi/3$ we have the simpler formula

$$\theta_s = 0.918\theta_w - 7.5 \times 10^{-3}\theta_w^3. \tag{3.3}$$

If the equation of the body shape $z = f(x, y)$ and the equation of the form of the shock wave $z_s = f_s(x, y)$ are specified, the following relations hold:

$$\sin\theta_w = \frac{1}{\sqrt{1 + f_x'^2 + f_y'^2}}, \quad \sin\theta_s = \frac{1}{\sqrt{1 + f_{sx}'^2 + f_{sy}'^2}} \tag{3.4}$$

and the formula for the radiation heat flux to the body surface takes the form

$$q_w^R(\theta_s) = q_{w0}^R \cos^n\theta_s \tag{3.5}$$

Then the overall radiation heat flux to the body surface is

$$Q_R = \iint_S q_w^R\, dS = q_{w0}^R \iint_S \sin^n\beta\, dx\, dy, \quad \beta = \pi/2 - \theta_S \tag{3.6}$$

In [7,9] a generalization to the three-dimensional case of the limiting formula for the radiation heat fluxes from a strongly radiating optically thin shock layer, previously obtained for axisymmetrical bodies, was employed.

Three-dimensional unsteady hypersonic flow of a radiating gas close to the windward surface of a wing of small length with a surface shape that varied with time was investigated in [26–28]. The use of the thin shock layer method [68] enabled a general solution to be obtained for the equations of gas dynamics.

The flow around a wing at the angle of attack α is considered in [27]. The compressed layer of gas close to the windward surface of the wing is optically transparent. A small parameter ϵ, equal to the ratio of the densities in the intense shock wave, is introduced. For flow around a wing with a shock wave, attached to the leading edge, the required gas-dynamic functions are represented in the form of expansions in powers of ϵ. When taking radiation transfer into account, the shock layer is considered to be locally one-dimensional. Then, using the well-known solution of the radiation-transfer equation [58] and neglecting radiation from the wing surface, the following expression is obtained for the local radiation heat flux to the wing [27]:

$$q_R(x, z) = \rho_\infty v_\infty^3 \sin^3\beta\, Q(x, z)$$

$$Q(x, z) = \frac{B}{2(n+4)} \int_{\chi_b}^{x} \frac{[1 + B(x - \chi)]^{-(n+5)/(n+4)}}{1 - (x - \chi)S_{zz}(\chi, z, t)}\, d\chi$$

$$B = \frac{8\sigma T_\infty^4 \tau A^4 \mu_M^{n+4}}{\rho_\infty v_\infty^3 \sin\beta}(n + 4) \tag{3.7}$$

Here σ is the Stefan–Boltzmann constant, A and n are coefficients of the approximation of the Planck absorption coefficient, μ_M is the molecular weight of the gas, $S(\chi, z, t)$ is a function of the form of the shock wave, β is the angle between the normal to the shock wave and the vector of the free stream velocity v_∞, τ is the optical thickness of the compressed layer, (x, y, z) are the Cartesian coordinates in a system of coordinates attached to the wing (x is the longitudinal coordinate), the thickness of the body measured from the plane $y = 0$ is small, and χ is the abscissa of the point of intersection of the trajectory of a given gas particle with the shock wave. When writing these equations it was assumed that on the shock wave $\chi = x$. The values of χ_b for trajectories lying on the wing surface are found assuming that the shape of the leading edge of the wing in plan $z = z_b(x)$ is independent of time [27]. If the shock wave is only attached to the top of the wing and is detached from the edge, all the trajectories on the wing pass through the top and over the whole wing surface $\chi_b = 0$.

There is a class of particular accurate solutions corresponding to the following shape of the surfaces of the body and the shock wave [27]:

$$y = F(x, z, t) = -\frac{f(x, t) z^2}{2}$$
$$y_s = S(x, z, t) = G(x, t) - \frac{f(x, t) z^2}{2}, \quad \chi_b = 0 \tag{3.8}$$

The case $f(x, t) = b/x$ corresponds to steady-state flow around a conical wing of unchanged shape with a transverse curvature in the plane of symmetry ($z = 0$) equal to ($b/x \operatorname{tg}\beta$). The radiation heat flux, according to (3.7) and (3.8) is constant along the span, which is a consequence of the constancy of the thickness of the compressed layer

$$Q(x) = \frac{B}{2(n+4)} \int_0^x \frac{[1 + B(x - \chi)]^{-(n+5)/(n+4)}}{(1-b)\chi + bx} \chi d\chi \tag{3.9}$$

The flow in the neighbourhood of a plane of symmetry on the windward side of a conical body at an angle of attack α was considered in [28]. Suppose that in a Cartesian system of coordinates (xyz) attached to the body, the plane of symmetry corresponds to $z = 0$, and the equation of the cross-section of the body in this plane is $y_b = x \operatorname{tg}\varphi$. The principal radii of curvature of the surface in the $z = 0$ plane are

$$R_1 = \infty, \quad R_2 = R_0 \xi \operatorname{tg}\varphi \sec\varphi, \quad \xi = x \sec\varphi.$$

The thickness of the compressed layer and the radiation heat flux then depend on the universal coordinate as follows:

$$G(X) = \frac{\operatorname{tg}(\varphi + \alpha)}{B} \int_0^X \frac{(1 + X - \chi)^{-1/(n+4)}}{(1 - C)\chi + CX} \chi d\chi$$
$$Q(X) = \frac{1}{2(n+4)} \int_0^X \frac{(1 + X - \chi)^{-(n+5)/(n+4)}}{(1 - C)\chi + CX} \chi d\chi \tag{3.10}$$
$$C = R_0^{-1} \operatorname{tg}(\varphi + \alpha) \operatorname{ctg}\varphi, \quad X = B\xi \sin^{2(n+4)}(\varphi + \alpha) \sec(\varphi + \alpha)$$

In a number of special cases, Eqs (3.10) reduce to the results obtained earlier by methods which can only be used for special forms of flow [58]. The set of expressions for the radiation heat fluxes to three-dimensional bodies is so far limited to these analytical formulas.

4. FUNDAMENTAL FORMULAS FOR CALCULATING THE TOTAL DRAG OF THREE-DIMENSIONAL BODIES

To calculate the wave-drag C_B and the total drag C_D we will consider a hypersonic flow of gas around a three-dimensional body at zero slip angle. We will again consider a Cartesian system of coordinates xyz attached to the body and denote the unit vectors of the Cartesian system of coordinates by **i**, **j**, **k**. We will introduce a unit vector of the normal n to an infinitely small element of the wetted surface dS_w (the positive direction of the normal is on the side from the surface) and the unit vector **t** tangential to dS_w and directed along the flow velocity at the specified point. As a result, the overall aerodynamic drag C_D is given by the expression [70]

$$C_D = \frac{2D}{\rho_\infty v_\infty^2} = \frac{1}{S} \iint_{S_w} [-C_p(\mathbf{n} \cdot \mathbf{k}) + C_f(\mathbf{t} \cdot \mathbf{k})] \, dS_w = C_B + C_{xf} \tag{4.1}$$

Here D is the total drag of the body, $q_\infty = \rho_\infty v_\infty^2/2$ is the free stream velocity head and C_p and C_{xf} are the pressure coefficient and the coefficient of frictional resistance of the body, respectively.

The pressure coefficient C_p is obtained from Newton's formula [31, 49, 61, 62]

$$C_D = \frac{1}{S} \iint_{S_w} \left[\frac{2}{(1 + f_x'^2 + f_y'^2)} + C_f \sqrt{f_x'^2 + f_y'^2} \right] dx \cdot dy \tag{4.2}$$

We will take into account the fact that the local coefficient of friction C_f can be expressed in terms of C_f^i, $i = 1, 2$ from (2.9) as follows:

$$C_f = ((C_f^1)^2 + (C_f^2)^2)^{1/2} \tag{4.3}$$

Then, using (2.9) for C_f^i we obtain an expression for the coefficient of frictional resistance C_{xf}.

We will assume that in a Cartesian system of coordinates (x, y, z) the Y axis is directed vertically upwards, in which case we can obtain an expression for the lift force coefficient C_L [2]

$$\begin{aligned} C_L &= \frac{L}{q_\infty S} = \frac{1}{S} \iint_S C_p(\mathbf{nj})\, dS \\ C_L &= \frac{2}{S} \iint_S \frac{f_y'\, dxdy}{1 + f_x'^2 + f_y'^2} \end{aligned} \tag{4.4}$$

Here L is the lift force of the body.

5. RULE OF EQUIVALENCE OR AREAS FOR THE WAVE DRAG, THE FRICTION DRAG AND THE RADIATION AND CONVECTIVE HEAT FLUXES

The design of shuttle-like hypersonic spacecraft raises the problem of investigating the integral characteristics of complex three-dimensional configurations consisting of "wing-fuselage" combinations [70]. For flows with relatively low hypersonic velocities there is a well-known area rule [68]. Using this rule one can estimate the effect of interference of the fuselage and wing on the aerodynamic forces for important practical configurations without having to make a detailed investigation of the flow pattern. A similar theorem is given in [44–46] for the wave drag of a blunt body at zero angle of attack in a hypersonic flow. An important limitation of this rule is the fact that the contours of the cross-sections of comparable bodies should not have corner points on those parts which protrude outside the region bounded by the shock wave.

A generalization of the equivalence rule for the wave drag in the case of steady and unsteady three-dimensional inviscid flow around thick bodies is given in [34]. This generalization is based on an averaging procedure over the angular variable of a cylindrical system of coordinates, which holds for bodies close to axisymmetrical. Numerical solution of a fairly wide range of problems has shown [34] that this generalized equivalence rule is applicable for essentially non-axisymmetrical configurations.

The hypersonic area rule can be combined with the law of similitude for flow around blunt thin bodies [68], as a result of which the flow around the body in appropriate dimensionless variables will be defined by two dimensionless parameters—the hypersonic parameter of similitude and a parameter which characterizes the blunting, and also one dimensionless function which expresses the variation of the area of the transverse cross-section.

An area rule was derived in [54, 67, 78, 79] for the change in the normal force and aerodynamic quality L/D of a triangular wing, flying at the angle of attack with a hypersonic velocity, as a result of adding to it, on the compression side of the flow, a conical body of arbitrary shape. Only small angles of attack are considered. It is suggested that the conical region of subsonic flow in which the body is placed occupies a small part of the wing. A generalization of the equivalence rule to non-conical bodies is obtained in [55]. A generalization of this rule to the case of the flow around a triangular wing for any sweepback angles and any angles of attack when the shock wave is attached to the leading edge is given in [67].

Area rules for three-dimensional bodies close to axisymmetrical have been proved in hypersonic gas dynamics. These rules are that the drag [34, 39, 49], the heat-transfer coefficient and the loss of mass [7] of a three-dimensional body are equal to the analogous quantities of an axisymmetrical body having the same distribution of the cross-section area along the axis. Note that in certain cases the area rule holds for bodies which depart considerably from axial symmetry [80].

A generalization of the area rule for other integral quantities in three-dimensional flows is derived in [6]. To calculate its range of applicability, heat fluxes in the case of flow at zero angle of attack around triaxial semi-ellipsoids with semiaxes a, b and c along the coordinate lines x, y, and z are considered in [7].

It follows from calculations that even when there is a considerable departure from axial symmetry ($b/a = 11$) the difference in Q_R reaches 30–40% in all [7].

The area rules considered above for quantities that are integral over the surface of the body can considerably facilitate the solution of the variational problems for three-dimensional bodies discussed above.

In conclusion we wish to thank G. A. Tirskii and Mrs I. G. Brykina for discussing the problems involved in calculating convective heat transfer and for a number of useful suggestions.

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Translated by R.C.G.